11.4 SIGNAL ENHANCEMENT BY TIME-FREQUENCY PEAK FILTERING [0]

11.4.1 Signal Enhancement and Filtering

Time-frequency peak filtering (TFPF) may be regarded as an unconventional alternative to the filtering methods described earlier in this chapter [1]. The signals considered are assumed to be sums of arbitrary numbers of band-limited non-stationary components in additive noise. For high SNR situations many signal processing algorithms work well but most perform poorly when SNR decreases below a given threshold [2]. In this case, signal enhancement algorithms are required to improve the SNR by reducing the distorting effects of noise. To this effect, both adaptive and fixed methods have been developed in the case of non-stationary signals in noise. Adaptive techniques are generally superior in performance to fixed methods, but they perform poorly in certain conditions, such as filtering of a non-stationary signal whose spectral content changes rapidly with time. For example, filters designed using a *least-mean-square* (LMS) approach may not adapt quickly enough to track the rapidly changing signal due to the delayed convergence of the algorithm. Further, adaptive methods require that the structure of the filter (such as the number of the taps) and an estimate of SNR be imposed for optimal performance. This is often not possible as assuming a model may lead to suboptimal results and even to erroneous conclusions about the signal. This suggests the need for a more general filtering method when the SNR is low and the underlying signal statistics vary rapidly with time.

The TFPF method is based on encoding the noisy signal as the IF of a unit amplitude frequency modulated (FM) analytic signal. The instantaneous frequency (IF) of the analytic signal is then estimated using standard time-frequency peak detection methods [2] to obtain an estimate of the underlying deterministic signal. For some signals, TFPF using a windowed WVD results in a significant enhancement of signals for SNR as low as −9 dB.

11.4.2 Time-Frequency Peak Filtering

11.4.2.1 Background and Definitions

Let us consider signals expressed as follows:

$$s(t) = x(t) + n(t) = \sum_{k=1}^{p} x_k(t) + n(t) \tag{11.4.1}$$

where $n(t)$ is an additive white Gaussian noise (WGN) and $x_k(t)$ are band-limited non-stationary deterministic components that may have overlapping frequency spectra. It is desired to recover the signal $x(t)$ given the observation of $s(t)$.

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The IF of an analytic signal, $z(t) = a(t)e^{j2\pi\phi(t)}$, is defined in Chapter 1 and reference [2]:

$$f_z(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} \tag{11.4.2}$$

where $\phi(t)$ is the instantaneous phase and $a(t)$ is the instantaneous amplitude of the analytic signal $z(t)$ which can be expressed as [3]:

$$z(t) = a(t)e^{j2\pi\int_{-\infty}^{t} f_z(\lambda)d\lambda} \tag{11.4.3}$$

Among the existing techniques for IF estimation [2], we adopt the method that estimates the IF by taking the peak of the signal's TFD for its simplicity of implementation. The WVD is a natural first choice as a TFD for peak filtering given that the other quadratic TFDs are simply smoothed versions of the WVD [3]. The WVD of the analytic signal $z(t)$ is defined (in Section 2.1.4) as:

$$W_z(t,f) = \int_{-\infty}^{\infty} z(t+\tau/2)z^*(t-\tau/2)e^{-j2\pi f\tau}d\tau. \tag{11.4.4}$$

For monocomponent FM signals, the WVD will produce a time-frequency representation of the signal exhibiting significant energy concentration around the signal's IF. When the signal's IF is linear, delta functions will appear at the positions of the IF providing a perfect signal IF estimate. The IF estimate is found by maximizing the WVD over frequency [2]; that is

$$\hat{f}_z(t) = \underset{f}{\operatorname{argmax}}[W_z(t,f)] \tag{11.4.5}$$

The IF estimate based on the peak of the WVD is unbiased and has variance approaching the Cramer-Rao lower bound for signals with linear IF laws in *additive* white zero-mean noise with moderate to high SNR [2]. However, as the order of the polynomial IF increases, the delta functions will be replaced by less peaky functions. The peak of these functions will lie away from the true IF, resulting in IF estimates which are biased. To remedy this, a windowed WVD is used, such that the signal IF is as close to linear as possible across the entire window length.

11.4.2.2 Basic Principle

Time-frequency peak filtering consists of a two step procedure whereby the signal to be filtered is first encoded as the IF of a unit amplitude FM modulated analytic signal. Then, the IF is estimated by taking the peak of a time-frequency distribution (TFD) to recover the filtered signal. This may be summarized as follows:

Step 1. Encode the noisy signal $s(t)$ via FM modulation as: $z_s(t) = e^{j2\pi\mu\int_0^t s(\lambda)d\lambda}$ where μ is a scaling parameter analogous to the FM modulation index.

Step 2. Estimate the peak of the WVD of the analytic signal $z_s(t)$:

$$\hat{x}(t) = \hat{f}_z(t) = \underset{f}{\operatorname{argmax}}[W_{z_s}(t,f)]/\mu$$

11.4.2.3 Properties

The properties of the encoding and IF estimation steps of TFPF are derived for the case of WVD only. The use of other TFDs will lead to slightly different properties.

Property 1: The encoding step converts the additive noise $n(t)$ to multiplicative noise $z_n(t)$ that modulates the signal component $z_x(t)$; that is

$$z(t) = e^{j2\pi\mu\int_0^t s(\lambda)d\lambda} = z_x(t)z_n(t) \tag{11.4.6}$$

where the encoded noise and deterministic signal components are given by

$$z_x(t) = e^{j2\pi\mu\int_0^t x(\lambda)d\lambda} \qquad \text{and} \qquad z_n(t) = e^{j2\pi\mu\int_0^t n(\lambda)d\lambda} \tag{11.4.7}$$

Proof: Equation 11.4.6 is obtained by a direct substitution of 11.4.1 into 11.4.3.

Property 2: The Wigner-Ville Spectrum of the signal $z_s(t)$ is given by :

$$\mathrm{WVS}_{z_s}(t,f) = E[W_{z_s}(t,f)] = \mathrm{WVS}_{z_n}(t,f) \underset{f}{*} W_{z_x}(t,f) \tag{11.4.8}$$

where $E[\cdot]$ is the expectation operator, $\mathrm{WVS}_{z_n}(t,f) = E[W_{z_n}(t,f)]$, and $\underset{f}{*}$ represents the convolution operation in the frequency domain.

Proof: This property follows from *property 1* and the direct application of the expectation operator to the WVD of the encoded signal $z_s(t)$.

This latter is given by:

$$E[W_{z_s}(t,f)] = \int_{-\infty}^{\infty} R_{z_n}(t,\tau)K_{z_x}(t,\tau)e^{-j2\pi f\tau}d\tau \tag{11.4.9}$$

where the time-dependent autocorrelation function of $z_n(t)$ is

$$R_{z_n}(t,\tau) = E[z_n(t+\tau/2)z_n^*(t-\tau/2)] = E[e^{j2\pi\mu\int_{t-\tau/2}^{t+\tau/2} n(\lambda)d\lambda}] \tag{11.4.10}$$

and the time-dependent bilinear product function of $z_x(t)$ is

$$K_{z_x}(t,\tau) = z_x(t+\tau/2)z_x^*(t-\tau/2) = e^{j2\pi\mu\int_{t-\tau/2}^{t+\tau/2} x(\lambda)d\lambda} \tag{11.4.11}$$

Equation (11.4.8) is then obtained by using the fact that the Fourier transform of a product in time is equivalent to the convolution in frequency.

Equation (11.4.8) shows that additive noise smears the encoded signal WVD, $W_{z_x}(t,f)$, through convolution. Therefore, the bias of TFPF is dependent on the encoded signal $z_x(t)$ as well as the shape of the encoded noise spectrum $\mathrm{WVS}_{z_n}(t,f)$. By restricting the shape of the encoded noise spectrum, a class of noise can be defined which does not introduce bias to the IF estimation. An example from this class is the WGN as will be seen next.

Property 3: The time dependent autocorrelation function of the encoded noise, $R_{z_n}(t,\tau;\mu)$, is equal to the characteristic function of $q(t,\tau) = 2\pi\int_{t-\tau/2}^{t+\tau/2} n(\lambda)d\lambda$; that is

$$R_{z_n}(t,\tau,\mu) = E[e^{j2\mu q(t,\tau)}] = \Phi_q(t,\tau,\mu) \tag{11.4.12}$$

where $\Phi_q(t,\tau,\mu)$ is the characteristic function of $q(t,\tau)$ defined by [4]:

$$\Phi_q(t,\tau,\mu) = E[e^{j\mu q(t,\tau)}] = \exp\left(\sum_{i=1}^{\infty}\frac{k_{qi}(t,\tau)(j\mu)^i}{i!}\right) \tag{11.4.13}$$

and $k_{qi}(t,\tau)$ is the ith cumulant of $q(t,\tau)$.

Proof: Equation 11.4.12 is obtained by forming the autocorrelation function of $z_n(t)$ in equation 11.4.7 and using the above definition of $q(t,\tau)$.

11.4.3 Accurate TFPF

Equation (11.4.8) suggests that in general, a bias in IF estimation is introduced by the time-frequency distribution of $z_x(t)$ (deterministic bias) and/or the noise (stochastic bias). In the case where the encoded signal $s(t)$ is composed of a deterministic signal $x(t)$ that is linear in time and embedded in stationary WGN $n(t)$, TFPF gives an unbiased estimate of the signal $x(t)$.

Proof: Consider the signal $s(t)$, given in (11.4.1), to be filtered using TFPF. For the case where $n(t)$ is stationary WGN, the ith cumulant of $n(t)$ is such that $k_{ni} = 0$ for $i \geq 3$ and $q(t,\tau)$ is Gaussian with $k_{qi} = 0$ for $i \geq 3$. Furthermore, if the noise is a zero-mean independent process, i.e. $R_n(\tau) = k_{n2}\delta(\tau)$, then [5, page 369]

$$k_{q1}(t,\tau) = 0 \qquad \text{and} \qquad k_{q2}(t,\tau) = 4\pi^2|\tau|k_{n2} \tag{11.4.14}$$

The characteristic function given in (11.4.13) becomes

$$\Phi_q(t,\tau,\mu) = e^{-2\pi^2\mu^2|\tau|k_{n2}} \tag{11.4.15}$$

Taking the Fourier transform of this expression gives

$$E[W_n(f,t)] = \frac{4\pi^2 k_{n2}\mu^2}{(2\pi^2 k_{n2}\mu^2)^2 + (2\pi f)^2} \tag{11.4.16}$$

This shows that the encoded noise spectrum is low-pass with a maximum at the frequency $0Hz$. Hence, WGN will not introduce any bias to the estimate of the IF. By replacing this last expression in (11.4.8) we obtain:

$$W_{z_s}(t,f) = W_{z_x}(t,f) \underset{f}{*} \frac{4\pi^2 k_{n2}\mu^2}{(2\pi^2 k_{n2}\mu^2)^2 + (2\pi f)^2} \tag{11.4.17}$$

This expression shows that the bias in the IF using the peak of $W_{z_s}(f,t)$ could only come from $W_{z_x}(t,f)$. For the case where the signal $x(t)$ is linear in time; that is $x(t) = \alpha t + C$, where α and C are constants, equation 11.4.17 becomes

$$W_{z_s}(t,f) = \delta(f - x(t)) \underset{f}{*} \frac{4\pi^2 k_{n2}\mu^2}{(2\pi^2 k_{n2}\mu^2)^2 + (2\pi f)^2} \tag{11.4.18}$$

$$= \frac{4\pi^2 k_{n2}\mu^2}{(2\pi^2 k_{n2}\mu^2)^2 + (2\pi f - 2\pi x(t))^2} \tag{11.4.19}$$

The delta function ensures that the peak of this function occurs at $x(t)$. Therefore if the signal $x(t)$ is linear in time and embedded in stationary WGN, TFPF gives an unbiased estimate.

Equation 11.4.17 shows that in the general case where the signal $x(t)$ is a nonlinear function of time, the WVD-based TFPF is biased, requiring an appropriate windowing of the data. The window is chosen such that the signal within this window behaves almost linearly [6] (see Section 11.4.4). In the special case where the signal $x(t)$ is a finite-order polynomial in time, the deterministic bias can be completely eliminated if the WVD is replaced by the polynomial WVD (PWVD) of an appropriate order since the PWVD exhibits delta functions along the IF law for polynomial FM signals [7].

11.4.4 Discrete-Time Algorithm for TFPF

The implementation of TFPF using the windowed WVD requires both signal scaling before encoding to prevent aliasing, and the selection of the window length for reduced bias. These two aspects are discussed next.

11.4.4.1 Signal Scaling

FM modulation of un-scaled discrete time signals can lead to aliasing which produces discontinuities in the estimated IF at the frequency boundaries of the time-frequency plane. This is avoided by amplitude scaling of the noisy signal before frequency encoding. Without loss of generality and unless otherwise specified we assume that the signal $s(t)$ is sampled at a normalized sampling frequency of $1Hz$.

The scaled signal, $s_c(m)$, is obtained by using the following transformation.

$$s_c(m) = S[s(m)] = (a-b)\frac{s(m) - \min[s(m)]}{\max[s(m)] - \min[s(m)]} + b \tag{11.4.20}$$

where $S[.]$ is the scaling operator and the parameters a and b, which satisfy the constraint $.5 \geq a = \max[s_c(m)] > b = \min[s_c(m)] \geq 0$, are chosen to provide suitable frequency limits on the encoded signal. The operators max[.] and min[.] are the maximizing and minimizing functions respectively. The estimate of the desired signal, $\hat{x}(m)$, is recovered by an inverse scaling operation; that is

$$\hat{x}(m) = S^{-1}[\hat{x}_c(m)] = \frac{(\hat{x}_c(m) - b)(\max[s(m)] - \min[s(m)])}{a - b} + \min[s(m)] \quad (11.4.21)$$

where $\hat{x}_c(m)$ is the scaled signal obtained using TFPF on $s_c(m)$.

11.4.4.2 Reduced-Bias Window Length Selection

The bias-variance tradeoff is a key in the practical implementation of TFPF with the windowed WVD. Bias reduction requires a small window length to minimize the non-optimal nature of the WVD for higher than quadratic phase signals. On the other hand, variance reduction is achieved by increasing the window length to provide the local estimate with more information. To reduce the variance of the estimate while maintaining bias performance it becomes necessary to increase the sampling rate. Thus there is a tradeoff between estimator bias and sampling frequency which results in a relationship between window length and bias, for a given sampling frequency. Results relating to TFPF window lengths are derived in [6]. The basic results for window length τ_w, sampling frequency f_s and maximum value of IF, f_p are given below. For the case of signal estimation:

$$\tau_w \leq \frac{0.634 f_s}{\pi f_p} \quad (11.4.22)$$

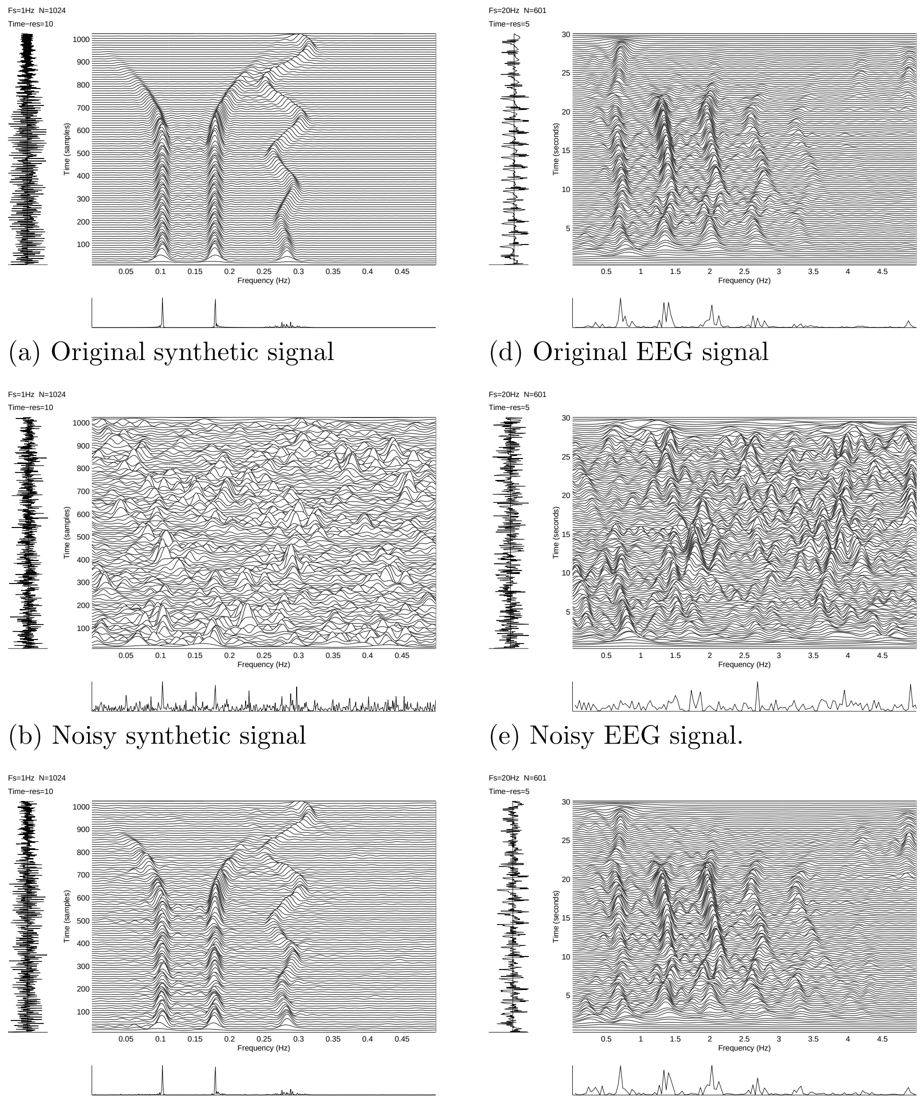
This equation gives the maximum window length as a function of maximum signal IF and sampling frequency. In a typical application a specified window length is required for a given SNR. The signal is sampled at a high enough rate to satisfy (11.4.22).

11.4.4.3 The Iterative TFPF Algorithm

In the first application, TFPF may not remove as much additive noise as desired. If this situation occurs, reapplication of the procedure to the filtered signal is recommended. This leads to a 3 step iterative process:

1. Scale and encode noisy signal.
2. Apply TFPF to yield a signal estimate $\hat{x}(t)$.
3. If $\hat{x}(t)$ contains substantial noise, go to step 1; else terminate the process.

Simulations demonstrate the convergence of the repeated scheme to a stable estimate $\hat{x}(t)$ [1].



(a) Original synthetic signal
(d) Original EEG signal
(b) Noisy synthetic signal
(e) Noisy EEG signal.
(c) Enhanced synthetic signal
(f) Enhanced EEG signal

Fig. 11.4.1: B-distributions of a synthetic signal (left column) and an EEG signal (right column), showing the original signal (top row), and the noisy signal before enhancement (middle row) and after enhancement (bottom row).

11.4.5 Examples and Results

Example 1 (A Multicomponent signal in WGN) Let us consider a multicomponent signal, $x(m)$, expressed by:

$$x(m) = \begin{cases} 0.85\sin(0.055m + 3.75\times10^{-4}\sin(0.000625m)m) + \sin(0.035m) \\ +\sin(0.020m), & 0 \le m < L/2 \\ 0.85\sin(0.055m + 3.75\times10^{-4}\sin(0.000625m)m) \\ +2(1-m/L)\sin(0.035m + 2.75\times10^{-11}(m-L/2)^3) \\ +2(1-m/L)\sin(0.020m - 2.75\times10^{-11}(m-L/2)^3), & L/2 \le m < L \end{cases}$$

where the data length L is taken as 32768 data points. For a time-frequency illustration of this signal, the B-distribution (BD) with smoothing parameter $\beta = 0.01$ is shown in Fig. 11.4.5(a).[1] White Gaussian noise was added to the above signal giving an SNR of -9 dB; the BD of the noisy signal is shown in Fig. 11.4.5(b). The windowed WVD peak filter was then implemented to recover $x(m)$ from the noisy signal. A window length of 15 data points was chosen to satisfy the window length constraints given in (11.4.22). Fig. 11.4.5(c) shows the clean recovery of the signal after three TFPF iterations. Note that the WVD is used as the vehicle for signal recovery while the B-distribution is used only for presentation of the results.

Example 2 (Newborn EEG data in WGN): Fig. 11.4.5(d) shows a time-frequency representation of a real newborn EEG signal using the B-distribution with $\beta = 0.01$. WGN is then added to the signal at $SNR = -9$ dB. The noisy signal in Fig. 11.4.5(e) shows that the time-frequency patterns of the EEG signal are not clearly visible. Using a window length of 20 data points, four iterations of the TFPF were used to recover a cleaner signal. The filtered signal in Fig. 11.4.5(f) demonstrates the efficiency of TFPF.

11.4.6 Summary and Conclusions

TFPF is a tool for signal enhancement, applicable to a large class of signals if the windowed WVD TFPF is used for reduced bias. This class includes those signals which may be represented as a sum of band-limited non-stationary processes in additive WGN. Testing on simulated and real data indicates that the method significantly enhances signals of this class by filtering out most of the additive noise. Further details of the time-frequency peak filtering method are provided in [8].

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